

Modelling of laboratory data of bi-directional reflectance of regolith surface containing Alumina

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Abstract: Bidirectional reflectance of a surface is defined as the ratio of the scattered radiation at the detector to the incident irradiance as a function of geometry. The accurate knowledge of the bidirectional reflection function (BRF) of layers composed of discrete, randomly positioned scattering particles is very essential for many remote sensing, engineering, biophysical applications and in different areas of Astrophysics. The computations of BRF's for plane parallel particulate layers are usually reduced to solve the radiative transfer equation (RTE) by the existing techniques. In this work we present our laboratory data on bidirectional reflectance versus phase angle for two sample sizes of 0.3 and 1 μm of Alumina for the He-Ne laser at 632.8 nm (red) and 543.5nm(green) wavelength. The nature of the phase curves of the asteroids depends on the parameters like- particle size, composition, porosity, roughness etc. In our present work we analyse the data which are being generated using single scattering phase function i.e. Mie theory considering particles to be compact sphere. The well known Hapke formula will be considered along with different particle phase function such as Mie and Henyey Greenstein etc to model the laboratory data obtained at the asteroid laboratory of Assam University.

Keywords: comets: general – dust, extinction – scattering – polarization

1 Introduction

The study of the light scattering properties of powdered materials is known to be an important tool for characterizing the physical and compositional properties of asteroids. It is well conceived that asteroids are covered with finely grained materials known as regolith layers (Hapke 2005). Hence it is imperative that laboratory based experiments on the asteroid analogues can be compared with the in situ data as well as the theoretical models can also be tested. As the phase angle approaches to zero, the brightness of asteroids increases very rapidly, the phenomenon is termed as opposition effect. The various physical parameters like particle size, porosity, surface roughness, thickness of the layer etc are very important and being studied in laboratory by many authors such as Kamei et al. (1999), Kaasalainen (2003) and Nelson et al. (2000). A large number of literature is available on the physical interpretation of opposition effect based on shadowing and coherent backscattering (Hapke 2002, Shkuratov et al. 2002). But it is difficult to explain with the theoretical models how the opposition effect depend on physical parameters.

At large phase angles all the physical parameters cannot be studied efficiently. In spite of that certain very important properties like composition, grain size, grain shape etc can be studied. The most widely used formula for describing the scattering of light from a particulate surface is the Hapke formulae (Hapke 2005) and Lumme & Bowel formula (1981). It requires at least three unknown parameters, amongst them two

become irrelevant for large phase angles. Recently Hapke et al 2009, compare the ability of several radiative transfer models to describe the scattering behavior measured over a wide range of phase angles. Shepard and Helfenstein (2007) studied bidirectional reflectance function for 14 different samples including 4 Al_2O_3 samples over a phase angle varied from 3° to 130° . Piatek et al. (2004), measured the variation of reflectance as the phase angle varied from 0.05° to 140° for particle size ranges from smaller to larger than the wavelengths.

In a preliminary work with alumina sample (Deb 2010) for zero tilt and observation wavelength of 632.8 nm, the phase curve was satisfactorily fitted with Hapke formula and Mie theory by varying absorption coefficient k . Here, we have included more experimental data at two different particle size of samples (0.3 micron and 1 micron) and observation wavelengths (632.8 nm and 543.2 nm) for different tilt angles to study the theoretical behavior in more detail. In the present work the photometric data at large phase angles for the plane surface of powdered alumina (Al_2O_3) with 0.3 micron and 1 micron average particle diameter at the above wavelengths have been generated. In the present analysis we have considered Mie theory (single particle scattering) i.e. the particles are compact and spherical in shape. We use Mie theory with Hapke formula in Henyey-Greenstein phase function to theoretically calculate bidirectional reflectance and model it with the laboratory data thus obtained.

2 Instrumentation and sample

The experiment was carried out with the help of a goniometric device at the department of Physics, Assam University, Silchar, India. It consists of two metal arms having a common horizontal axis of rotation. The sample surface is placed at the axis of rotation of the arms with the help of three translation stages. A miniature goniometer acts as a tilting device to the sample. The two arms can be rotated by $\pm 90^\circ$ from the zenith direction and a tilt up to $\pm 20^\circ$ can be given to the sample from the horizontal position perpendicular to the plane of scattering. We have used He-Ne laser of red and green wavelengths as the source of light and the CCD camera as the detector. The sample is placed at the common intersection of the axis of rotation and axes of the source and detector. A diffuser was placed in front of the CCD to reduce the laser speckles produced by the coherent laser beam on scattering from the rough surface. It is evident that the diffuser incorporates some uncertainty in emergent angle, to address this criteria we calculated the solid angle as well as the uncertainty in the emergent angle as 0.028 Steradian and ± 0.32 degree respectively. The intensity at any point on an illuminated area, along a given direction, is defined as the power radiated per unit projected area of illumination, to the direction under consideration, per unit solid angle. In this case, the solid angle feature can be neglected since it is a constant for a particular instrument and finally gets canceled out while taking the ratio.

The sample used in the present work is of powdered alumina (Al_2O_3). We have used two sample of different average value of diameter, $0.3\mu\text{m}$ and $1\mu\text{m}$. Here after we shall call $0.3\mu\text{m}$ as sample-I and $1\mu\text{m}$ as sample-II. At the initial stage the surface roughness is quite high. For the preparation of the smooth surface, the sample surface was pressed by a smooth metal plate so that the sample surface takes its smoothness.

3 Data collection and reduction

The tilt angle of the sample was set fixed at 0° first, then varied from $\pm 2^\circ$ to $\pm 20^\circ$ at every 2° interval. For simplicity of the theoretical models, we took tilt angles $0^\circ, 10^\circ, 20^\circ$. The detector angles (e) were kept fixed at 45° and 63° (the sign is positive accounts for forward scattering ref Fig.1) from the Zenith. The angle of incidence (i) was varied from 0° to 63° in steps of 9° . Hence the phase angles also varied from 45° to 126° . The detector readings were collected at every new angle of incidence, and the images of the sample surface were recorded in the form of FITS image. As the field of view of the detector was larger than the laser spot, geometrical correction ($\cos i / \cos e$) was necessary to calculate the intensity values from the detector counts. Corrections for the background were also done for each observation. The reflectance values were calibrated by using BaSO_4 (a standard Lambert surface) at incidence angle 0° and detector angle 45° .

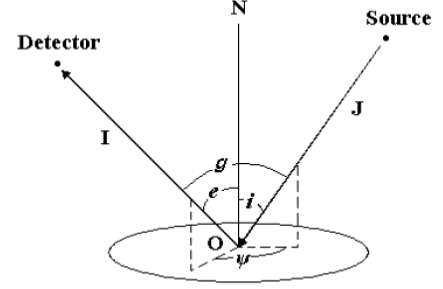


Figure 1: Schematic diagram of bidirectional reflectance.

4 Theory

The bi-directional reflectance r (i.e.g) is defined as the ratio of the reflected intensity (I) to the incidence irradiance (J) measured for alumina sample is shown in Fig-1 (which shows the experimental set up).

When a beam of collimated light is incident on a rough surface, the Fresnel laws of reflection are not obeyed by the reflected light as it gets scattered along all directions throughout the upper hemisphere. The condition $g = i + e$, holds, if the planes of emergence and incidence coincides ($\psi = 0$ or 180°) and the tilt angle becomes 0° . In the present study for other tilt angles viz, $10^\circ, 20^\circ$, the phase angle $g \neq i + e$. The intensity of the scattered beam depends on these three angular parameters. The bidirectional reflectance ' r ' as a function of i , e and g is given by

$$r(i, e, g) = I(i, e, g) / J \quad (1)$$

The interrelation among the angle of incidence i , detector angle e , the phase angle g and the tilt angle ϕ is given by,

$$\cos g = \cos i \cdot \cos e + \sin i \cdot \sin e \cdot \cos \phi \quad (2)$$

4.1 Mie Theory

Mie theory is a single particle light scattering theory, which was theoretically derived for the solution of light scattered from smooth and homogeneous sphere of any size (van de Hulst 1957). It depends on the complex refractive index (n, k) and the size parameter $X = 2\pi a / \lambda$, where a , λ are radius and wavelength of the light respectively. It is true fact that Mie theory is applicable only for a single and isolated spherical particle and not directly applicable when there are a number of particles in contact with each other, because, in that case multiple scattering between one particle to another comes across which makes the scattering behaviour complicated. But, the approach considered in this work demands for a 'single particle phase function'

into Hapke formula. The calculation of multiple scattering is done by Hapke formula independently. Also, the 'single particle phase function' of a isolated particle and a particle in regolith differs only by a little amount (e.g. Fig.1 of Hapke et al. 2009) which has been neglected in this study. To model the laboratory data of bidirectional reflectance, we use Mie theory to calculate single particle albedo ω and the asymmetry parameter ξ . It is hardly accepted that the particles of alumina are smooth and homogeneous spheres. But Pollack and Cuzzi (1980) suggested that the Mie theory may be used to calculate the scattering properties of equant irregular particles also, provided size parameter $X \leq 5$. In the present work the size parameters are 1.49 and 1.73 for sample-1 with red and green wavelengths which also justifies the above fact.

4.2 Hapke Model

This model describes the scattering of light from a particulate surface, which has been derived from the theory of radiative transfer. The Hapke formula mainly has three parameters, i.e. single particle scattering albedo ω , single particle phase function $p(g)$, opposition surge amplitude B_0 , opposition surge width h . But for larger phase angles $> 45^\circ$, effect of $B(g)$ can be neglected. The Hapke formula is given by, (Hapke 2002, 2005)

$$r(i, e, g) = (\omega/4\pi) \frac{\mu_0}{\mu_0 + \mu} [\{1+B(g)\}p(g) + H(\mu_0)H(\mu) - 1] \quad (3)$$

where, $\mu_0 = \cos i$ and $\mu = \cos e$ and $P(g) = (1 - \xi^2)/(1 + 2\xi \cos g + \xi^2)^{3/2}$, and $H(\mu_0) = (1 + 2\mu_0)/(1 + 2\gamma x)$, $H(\mu) = (1 + 2\mu)/(1 + 2\gamma x)$ and $\gamma = (1 - \omega)^{1/2}$.

It is evident that for brighter surface, the average photon is scattered higher number of times before emerging from the surface causing the directional effects to be averaged out and multiply scattered intensity distribution to closely approach the isotropic case. The exact numerical solution for high albedo surface was obtained by Chandrasekhar (1960). The comparison of exact and approximate solution for isotropic scattering has been shown by [Hapke-1 (1981), in Fig-3,4,5]. In the same paper Hapke compared the H-functions versus μ for several values of single scattering albedo for Chandrasekhar's exact solution with his approximation (Fig-2) and found that the two solutions agree to better than 3% every where. In actual practice it is seen that single scattering albedo $\omega = 1$, has never been achieved and a slight decrease in ω value significantly increases the agreement between exact and approximate solution. It is reported that by Hapke-1 (1981) that when $\omega = 0.975$, the error is only 0.7%. Therefore it is quite justified that we took Hapke formula for our present analysis. As the theory demands an arbitrary single particle phase function $P(g)$, we consider an empirical phase function i.e. Henyey-Greenstein phase function with one term. This introduces a new unknown parameter ξ , known as asymmetry parameter. It takes the value between -1 and +1. These asymmetry factor and single particle scattering albedo are being calculated from running Fortran Code

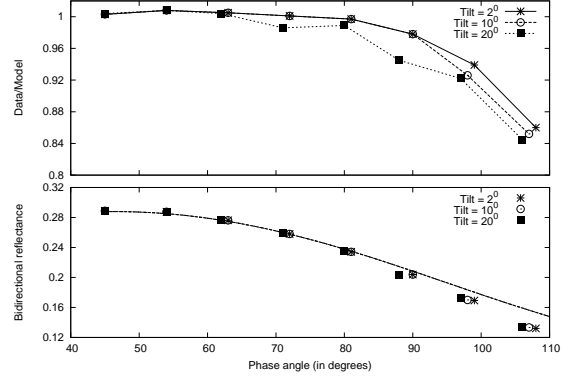


Figure 2: The upper panel shows the matching of Data:Model values ratio to 1. The lower panel gives the bidirectional reflectance vs phase angle for different tilt angles for sample-I at wavelength $\lambda = 632.8 \text{ nm}$ ($e=45^\circ$). The solid line in the lower panel represents the model.

on Mie theory, Published by Mishchenko et al. (1999), (available online at <http://www.giss.nasa.gov/~crmm>).

Therefore with the help of Mie theory in Hapke formula with Henyey-Greenstein phase function (Henyey & Greenstein 1941), we calculate approximate theoretical bidirectional reflectance of powdered alumina sample. In the next section in results and discussion, we show the nature of graphs obtained theoretically and compare this with experimentally obtained graphs.

5 Results and Discussion

The refractive index of alumina at 632.8nm is $n = 1.766$ (Gervais 1991) and absorption coefficient k is known to be very small. For the present fit our free parameter is k , we tried with different values of k and finally we found for our model the appropriate value of $k=0.00001$. Similarly for green laser of wavelength 543.5 nm, having $n=1.771$, the best fit value of k found to be 0.000001 which is comparable with earlier work, reported for tilt angle $=0^\circ$ by Piatek et al (2004).

Piatek et al. (2004) studied the absolute reflectance versus phase angle for alumina at different phase angles with average particle diameter \leq wavelength, the data thus reported is comparable with the present work for sample-I. In this work we have clearly showed how the Hapke model can be used to empirically fit the laboratory data not only for zero tilt angle but also for higher tilt angles (eg, 10° , 20° etc). Having said that there is a basic difference in our calculation of bidirectional reflectance with that of Piatek et al in their work they kept the angle of incidence fixed and varied angle of emergence while in this work we have two sets of fixed emergent angles 45° , 63° and angle of incidence varied from 0° to 63° .

For sample-II, the average particle size is greater than the wavelength of the laser source. In such condition we have found our asymmetry parameter ξ is posi-

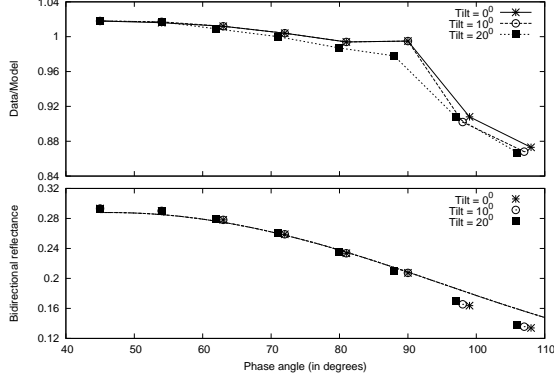


Figure 3: The upper panel shows the matching of Data:Model values ratio to 1. The lower panel gives the bidirectional reflectance vs phase angle for different tilt angles for sample-II at wavelength $\lambda = 632.8 \text{ nm}$ ($e=45^\circ$). The solid line in the lower panel represents the model.

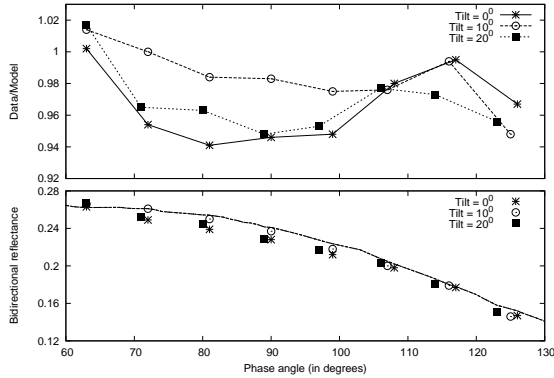


Figure 4: The upper panel shows the matching of Data:Model values ratio to 1. The lower panel gives the bidirectional reflectance vs phase angle for different tilt angles for sample-I at wavelength $\lambda = 543.5 \text{ nm}$ ($e=63^\circ$). The solid line in the lower panel represents the model.

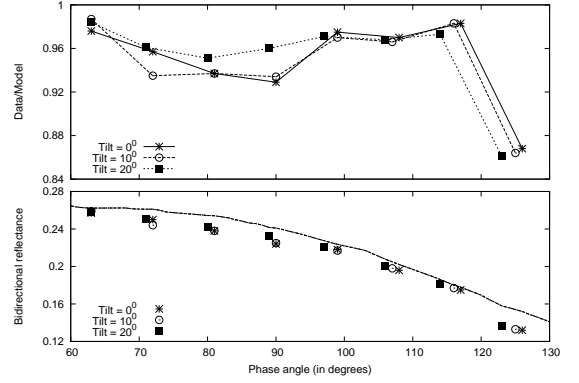


Figure 5: The upper panel shows the matching of Data:Model values ratio to 1. The lower panel gives the bidirectional reflectance vs phase angle for different tilt angles for sample-II at wavelength $\lambda = 543.5 \text{ nm}$ ($e=63^\circ$). The solid line in the lower panel represents the model.

tive which suggests that phase function is forward scattering, for phase angle ranges from 45° to 126° . This result is in accordance with other previously reported work which says for non opaque material in a powder the single scattering phase function is forward scattering. (Mishchenko 1994; Mishchenko & Macke 1997).

At present we are unable to show how our results is in accordance or in conflict with Shepard and Helfenstein (2007), as they tested the significance of Hapke photometric model, due to non availability of sufficient photometric data of Al_2O_3 at average particle diameter greater than wavelength of laser source.

It is quite obvious that fit to laboratory data of bidirectional reflectance by Hapke model would be better if we can relate the results with physical properties like porosity, roughness etc. The very fact that we have taken particles to be smooth spheres may incorporate certain uncertainties in modelling as the shape of the particles may be non spherical also.

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